New method of restoration of internal structure 3D bodies by means of projections which arrive from a computer tomograph.

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ABSTRACT

The problem of reconstruction of a three-dimensional body internal structure (factor of attenuation, body density) with help projections which arrive from a computer tomograph is one of most difficult of 3D x-ray computer tomography. Therefore creation of new general method of restoration of internal structure of the body, based on known schemes of calculation of Fourier's coefficients of two variables functions by means of projections which arrive from a computer tomograph is actual.
The work purpose: creation of a new general method of calculation $f(x,y,z)$ *in a kind*

$$T(x,y,z)=\sum_{k=-N}^{N}\sum_{\ell=-N}^{N}C_{k,\ell}(z)e^{i2\pi(kx+\ell y)},$$

where

$$C_{k,\ell}(z)=\sum_{s=1}^{M}C_{k,\ell}(z_s)h_s(z),$$

$h_s(z)$ *– basic splines (the order 1, 2, 3, …) with non-uniform knots of splines and with properties*

$$h_s(z_p)=\delta_{s,p},\quad 1\le s,p\le M,$$

$$\delta_{s,p}=1,\quad s=p;\quad \delta_{s,p}=0,\quad s\ne p;\quad s,p=1,2,...,$$

$C_{k,\ell}(z_s)=\int_0^1\int_0^1 f(x,y,z_s)e^{-i\cdot 2\pi(kx+\ell y)}dxdy$ *– Fourier's coefficients of function* $f(x,y,z_s)$ *at fixed value* $z=z_s$ *which find by any known method of the decision of a 2D x-ray computer tomography. For example, they can be found by a method of convolution of back-projections, direct Fourier's method, an algebraic method; a method based on use of Poisson's formula, by wavelets; method based on optimal approximation by piece-wise constants splines, the method based on use interlineations (the generalised algebraic method) etc.*
In work the theoretical substantiation of the offered method (an error ε_1 *of approach depending on* $\Delta=\max_{1\le s\le M-1}(z_{s+1}-z_s)$ *, and also from error* $\varepsilon_2=\max_{1\le s\le M}\left|C_{k,\ell}(z_s)-\tilde{C}_{k,\ell}(z_s)\right|$ *) is considered. Examples of application of the developed method on industrial process tomography are given.*
Authors thinks that proposed method can lead (after corresponding extension) to optimal algorithms the solutions of 3D computer tomography problems.

Keywords X-ray computer tomography, Fourier's coefficients, interflatation, interlineation, wavelets.

1 INTRODUCTION

The problem of restoration of spatial internal structure of a body (factor of attenuation, body density) on the known projections arriving from a computer tomograph, is one of most difficult problems of 3D a computer tomography. Besides, direct Fourier method (Gottlieb, Gustafsson, 1998) decisions of a two-dimensional problem of Radon's computer tomography in which basis the original method of calculation of Fourier's coefficients of two variables functions by means of the projections arriving from a computer tomograph lays is well-known. Authors do not know generalization of the specified method on a case of calculation of Fourier's coefficients of three variables functions

$$C_{p,q,r}=\int_0^1\int_0^1\int_0^1 f\left(x,y,z\right)e^{-i2\pi(px+qy+rz)}dxdydz,\ -N\le p,q,r\le N\ .$$

Use of such Fourier's coefficients allows to describe three-dimensional distribution of attenuation factors of $f\left(x,y,z\right)$ in the form of a trigonometrical polynom

$$S_N f\left(x,y,z\right)=\sum_{p=-N}^{N}\sum_{q=-N}^{N}\sum_{r=-N}^{N}\lambda_{p,q,r}C_{p,q,r}e^{i2\pi(px+qy+rz)},\ \lambda_{p,q,r}\in R\ ,$$

which is Fourier's sum at $\lambda_{p,q,r}=1,\ -N\le p,q,r\le N$; Fejer's sum at

$$\lambda_{p,q,r}=\left(1-\frac{|p|}{N+1}\right)\left(1-\frac{|q|}{N+1}\right)\left(1-\frac{|r|}{N+1}\right),\ -N\le p,q,r\le N\ ;$$

Rogozinsky's sum at $\lambda_{p,q,r}=\cos\left(\frac{p\pi}{2\left(N+1\right)}\right)\cos\left(\frac{q\pi}{2\left(N+1\right)}\right)\cos\left(\frac{r\pi}{2\left(N+1\right)}\right),\ -N\le p,q,r\le N$ and so on. Considering that the theory of approximation of functions by trigonometrical polynoms belongs to the most developed in the theory of approximation of many variables functions. Therefore the problem of generalization of direct Fourier's method on a three-dimensional case is actual.

For given time methods of a finding of unknown functions of three variables in the set sections by means of the projections arriving from a computer tomograph in the form of tomograms are well studied. But tomograms lay on system of parallel planes, and restoration of all function of three variables $f(x,y,z)$ can be made in the different ways. Most the general method of such restoration is the method based on use interflatation of functions (Lytvyn, Pershina, 2005). It consists in construction of such formulas, which on the set planes coincide in each point with the image of the corresponding tomogram. It allows to consider, that we can find the approached value of unknown function $f(x,y,z)$ in each point (x,y,z), and, thus, to find the approached value of its Fourier's coefficients $C_{p,q,r}(f)$. But for the approached finding of Fourier's coefficients $C_{p,q,r}(f)$ it is possible to use as well methods which are already used in a computer tomography, for example, a method of convolution of inverse projections, algebraic Fourier's method (Gordon, Bender, Herman, 1970), a method based on use interlineation of functions (the generalized algebraic method), a method based on wavelets (Lytvyn, Pershina, Nechuyviter, Litvin, Kulyk, 2009).

2 GENERAL METHOD OF CALCULATION $C_{p,q,r}$ ON THE BASIS OF METHODS OF RESTORATION OF FACTOR OF ATTENUATION ON SYSTEM OF PLANES $z=z_s,\ s=\overline{1,n}$

Let for functions $f(x,y,z_s),\ s=\overline{1,M}$ (generally speaking, unknown) their Fourier's coefficients which can be found by means of methods of a computer tomography (a method of convolution of inverse projections, an algebraic method, direct Fourier's method, a method based on interlineation of functions (the generalized algebraic method), a method based on wavelets, etc.) are known

$$C_{p,q}^{(s)}=\int_0^1\int_0^1 f(x,y,z_s)e^{-i2\pi(px+qy)}dxdy,\quad 0\le|p|\le N,\ 0\le|q|\le N\ .$$

Then for a finding

$$C_{p,q,r}=\int_0^1\int_0^1\int_0^1 f\left(x,y,z\right)e^{-i2\pi(px+qy+rz)}dxdydz$$

the method is offered

$$C_{p,q,r}\approx CS_{p,q,r}=\int_0^1 S_{p,q}(z,\{C_{p,q}^{(s)}\})e^{-i2\pi rz}dz\ ,$$

where

$$S_{p,q}(z,\{C_{p,q}^{(s)}\})=\sum_{s=1}^{M}C_{p,q}^{(s)}h_s(z),$$

$h_s(z)$ – basic functions (polynoms of degree $n-1$ or splines of order $m,\ 1\le m\le 3$) with properties

$$h_\mu(z_\nu)=\delta_{\mu\nu},\ \mu,\nu=\overline{1,M}\ .$$

Let's enter designations

$C_{p,q}(z)=\int_0^1\int_0^1 f(x,y,z)e^{-i2\pi(px+qy)}dxdy,\ \ 0\le|p|,|q|\le N$, $\tilde{C}^{(s)}_{p,q}, 0\le|p|,|q|\le N$ – Fourier coefficients, finding by one of the method the solution 2D tomography problem for the section of the body by planes $z=z_S,\ s=\overline{1,M}$ (by direct Fourier method or other methods mentioned above) and

$$\max_{0\le|p|,|q|\le N}\left|C^{(s)}_{p,q}-\tilde{C}^{(s)}_{p,q}\right|=O(\varepsilon_1),\ \varepsilon_1\to 0 .$$

Theorem 1. For an estimation of error $\left|C_{p,q}(z)-S_{p,q}(z,\{C^{(s)}_{p,q}\})\right|$ for a case when $S_{p,q}(z,\{C^{(s)}_{p,q}\})$ are a splines of order $m,\ 1\le m\le 3$, $\Delta=\max\limits_{1\le s\le n-1}(z_{s+1}-z_s),\ \forall f(x,y,z)\in C^{(0,0,m+1)}[0,1]^3$ the following is valid

$$\max_{0\le|p|,|q|\le N}\max_{0\le z\le 1}\left|C_{p,q}(z)-S_{p,q}(z,\{C^{(s)}_{p,q}\})\right|=O\left(\Delta^{m+1}\right)=O\left(\varepsilon_1\right),\ \forall f(x,y,z)\in C^{(0,0,m+1)}[0,1]^3 .$$

Proof follow from known approximate properties of the spline of order m .

For proving theorem 2 we used next statement

$$\max_{0\le|p|,|q|\le N}\max_{0\le z\le 1}\left|S_{p,q}(z,\{C^{(s)}_{p,q}\})-S_{p,q}(z,\{\tilde{C}^{(s)}_{p,q}\})\right|=O\left(\varepsilon_2\right),\ \varepsilon_2\to 0 .$$

Theorem 2. Let $\breve{C}S_{p,q,r}=\int_0^1 S_{p,q}(z,\{\tilde{C}^{(s)}_{p,q}\})e^{-i2\pi rz}dz$. Then for error $\left|C_{p,q,r}-\breve{C}S_{p,q,r}\right|$ next estimation is valid

$$\left|C_{p,q,r}-\breve{C}S_{p,q,r}\right|=O\left(\varepsilon_1\right)+O\left(\varepsilon_2\right),\ \varepsilon_1\to 0,\ \varepsilon_2\to 0 .$$

Proof. From theorem 1 and statement for $C_{p,q,r}$, $\breve{C}S_{p,q,r}$ we can write

$$\left|C_{p,q,r}-\breve{C}S_{p,q,r}\right|=\left|\int_0^1 C_{p,q}(z)e^{-i2\pi rz}dz-\int_0^1 S_{p,q}(z,\{\tilde{C}^{(s)}_{p,q}\})e^{-i2\pi rz}dz\right|=\left|\int_0^1[C_{p,q}(z)-S_{p,q}(z,\{\tilde{C}^{(s)}_{p,q}\})]e^{-i2\pi rz}dz\right|\le$$

$$\le\int_0^1\left|C_{p,q}(z)-S_{p,q}(z,\{C^{(s)}_{p,q}\})+S_{p,q}(z,\{C^{(s)}_{p,q}\}-S_{p,q}(z,\{\tilde{C}^{(s)}_{p,q}\})\right|dz\le$$

$$\le\int_0^1\left|C_{p,q}(z)-S_{p,q}(z,\{C^{(s)}_{p,q}\})\right|dz+\int_0^1\left|S_{p,q}(z,\{C^{(s)}_{p,q}\})-S_{p,q}(z,\{\tilde{C}^{(s)}_{p,q}\})\right|dz=O\left(\varepsilon_1\right)+O\left(\varepsilon_2\right),\ \varepsilon_1\to 0,\ \varepsilon_2\to 0 .$$

The theorem 2 is proved.

Then by this found Fourier's coefficients $\breve{C}_{p,q,r}$ it is possible to approach unknown function $f(x,y,z)$ in a kind

$$f(x,y,z)\approx F_{N,N,N}(x,y,z) ,$$

where

$$F_{N,N,N}(x,y,z)=\sum_{p=-N}^{N}\sum_{q=-N}^{N}\sum_{r=-N}^{N}\lambda_{p,q,r}\breve{C}_{p,q,r}e^{-i2\pi(px+qy+rz)} ,$$

Theorem 3. For $f\left(x,y,z\right)\in C^{m+1,m+1,m+1}$ the following estimation of an error is valid

$$\left|f(x,y,z)-F_{N,N,N}(x,y,z)\right|\le\left|f(x,y,z)-S_N f\left(x,y,z\right)\right|+\left|S_N f\left(x,y,z\right)-F_{N,N,N}(x,y,z)\right|\le O(\frac{1}{N^{m+1}})+(2N+1)^3 O(\varepsilon_1+\varepsilon_2) .$$

Proof. Here we used a known estimation $\max\limits_{x,y,z}\left|f(x,y,z)-S_N f\left(x,y,z\right)\right|=O(\frac{1}{N^{m+1}})$ and inequality

$$\left|S_N f\left(x,y,z\right)-F_{N,N,N}(x,y,z)\right|=\left|\sum_{p=-N}^{N}\sum_{q=-N}^{N}\sum_{r=-N}^{N}\lambda_{p,q,r}\left(C_{p,q,r}-\breve{C}_{p,q,r}\right)e^{-i2\pi(px+qy+rz)}\right|\le$$

$$\le\sum_{p=-N}^{N}\sum_{q=-N}^{N}\sum_{r=-N}^{N}\left|\lambda_{p,q,r}\right|\cdot\left|C_{p,q,r}-\breve{C}_{p,q,r}\right|\le(2N+1)^3 O(\varepsilon_1+\varepsilon_2) .$$

The theorem 3 is proved.

Thus, unknown function $f(x,y,z)$ in the offered method can be found approximately in the form of Fourier's sum on variables x,y with coefficients which are functions from z and are represented in the form of the corresponding splines $\breve{C}_{k,l}(z)$ constructed on a basis approximately found for each $z=z_s,\ s=\overline{1,M}$ of Fourier's coefficients.

3 METHOD OF CALCULATION OF FOURIER COEFFICIENTS OF TWO VARIABLES FUNCTIONS BY FINITE HAAR SUMS

The new method of calculation of Fourier coefficients of two variables functions $f(x,y)$ which is used at mathematical modelling in a computer tomography by Fourier's or Fejer's trigonometrical polynoms is offered. The method uses replacement of projections (integrals from function $f(x,y)$ along the set of lines, which cross object of research) the corresponding wavelet sums.
Let's use such auxiliary statements.

Theorem 4. At calculation of Fourier coefficients $CF_{k,l}=\int_0^1\int_0^1 f(x,y)e^{-i2\pi(kx+ly)}dxdy$ for $k=0,\ l=0;$ $k=0,\ l>0;\ \ k>0,\ l=0$ by projections equalities are fair

$$CF_{0,0}=\int_0^1\int_0^1 f(x,y)dxdy=\int_0^1\gamma_2(y)dy\ ,\ \gamma_2(y)=\int_0^1 f(x,y)dx\ ,$$

$$CF_{k,0}=\int_0^1\int_0^1 f(x,y)e^{-i2\pi kx}dxdy=\int_0^1\gamma_1(x)e^{-i2\pi kx}dx\ ,\ \gamma_1(x)=\int_0^1 f(x,y)dy\ ,$$

$$CF_{0,l}=\int_0^1\int_0^1 f(x,y)e^{-i2\pi ly}dxdy=\int_0^1\gamma_2(y)e^{-i2\pi ly}dy\ .$$

For $k<0,\ l<0$ we have $CF_{-|k|,0}=\overline{CF_{|k|,0}}$; $CF_{0,-|l|}=\overline{CF_{0,|l|}}$.

For $k\ge 1,\ l\ge 1$ and $k\ge l$: $\quad CF_{k,l}=I_1+I_2+I_3$,

$$I_1=l\int_0^1\frac{F_1(zl)e^{-i2\pi zl}dz}{k^2+l^2}\ ,\ I_2=\frac{k-l}{k^2+l^2}\int_0^1 F_2\left(l+z(k-l)\right)e^{-i2\pi[l+z(k-l)]}dz\ ,$$

$$I_3=l\int_0^1\frac{F_3(k+zl)e^{-i2\pi(k+zl)}dz}{k^2+l^2}\ ,\quad \text{where}\quad F_1(t)=\int_{-\frac{l}{k}t}^{\frac{k}{l}t} f\left(\frac{kt-lv}{k^2+l^2},\frac{lt+kv}{k^2+l^2}\right)dv\ ,$$

$$F_2(t)=\int_{-\frac{l}{k}t}^{\frac{k^2+l^2-lt}{k}} f\left(\frac{kt-lv}{k^2+l^2},\frac{lt+kv}{k^2+l^2}\right)dv\ ,\ F_3(t)=\int_{\frac{1}{l}\left[-k^2-l^2+kt\right]}^{\frac{1}{k}\left[k^2+l^2-lt\right]} f\left(\frac{kt-lv}{k^2+l^2},\frac{lt+kv}{k^2+l^2}\right)dv\ .$$

Similar formulas for other values of indexes k and l use other designations for projections (instead of F_μ designations G_μ , ϕ_μ , ω_μ , $\mu=1,2,3$). We omit them.

Let's notice, that for values $k,\ l$ satisfying to conditions $k,\ l\le -1$, equalities $CF_{-k,-l}=\overline{CF_{k,l}}$; $CF_{-k,l}=\overline{CF_{k,-l}}$, $\overline{\alpha+i\beta}:=\alpha-i\beta$ are used. Written above function F_μ , G_μ , ϕ_μ , ω_μ , $\mu=1,2,3$ are the projections received by integration of function $f(x,y)$ along straight lines, crossing square $[0,1]^2$ and pass in parallel direct $kx+ly=t$. In practice the specified experimental data can be received by computer tomograph for a discrete set of values of variable t .

The method consists in replacement of functions $\gamma_1(x),\ \gamma_2(y),\ \ F_1(zl),\ \ F_2\left(l+z(k-l)\right),$ by Haar's wavelet sums and in exact calculation of the received integrals. It is possible to use except Haar's wavelets as well others wavelets.

Theorem 5. For the approached calculation of Fourier's coefficients by finite Haar's sum wavelets and a discrete set of projections we have formulas

$$CF_{0,0}\approx CF_{0,0,M}=\frac{1}{M}\sum_{p=1}^{M}\gamma 1_p=\frac{1}{M}\sum_{q=1}^{M}\gamma 2_q\ ,\ CF_{k,0}\approx CF_{k,0,M}=\sum_{p=1}^{M}\gamma 1_p e^{-i\frac{p}{M}}\ ,$$

$$CF_{0,l}\approx CF_{0,l,M}=\sum_{q=1}^{M}\gamma 2_q e^{-i\frac{q}{M}}\ ,$$

$$CF_{k,l}\approx CF_{k,l,N}=I_{1,k,l,N_1}+I_{2,k,l,N_2}+I_{3,k,l,N_3},\quad k\ge l\ge 1\ ,$$

$$I_{1,k,l,N_1}=\frac{l}{k^2+l^2}\left(\frac{e^{-i2\pi\frac{l}{N_1}}-1}{-i2\pi l}\right)\sum_{q=0}^{N_1-1}\tilde{F}_1\left(\frac{z_q+z_{q+1}}{2}\right)e^{-i2\pi\frac{q}{N_1}l},\ \tilde{F}_1(z)=F_1(zl),\ z_q=\frac{q}{N_1},$$

$$I_{2,k,l,N_2}=\frac{k-l}{k^2+l^2}\left(\frac{e^{-i2\pi\frac{k-l}{N_2}}-1}{-i2\pi(k-l)}\right)\sum_{q=0}^{N_2-1}\tilde{F}_2\left(\frac{z_q+z_{q+1}}{2}\right)e^{-i2\pi\frac{q}{N_2}(k-l)},\ \tilde{F}_2(z)=F_2(l+z(k-l)),\ z_q=\frac{q}{N_2},$$

$$I_{3,k,l,N_3}=\frac{l}{k^2+l^2}\left(\frac{e^{-i2\pi\frac{l}{N_3}}-1}{-i2\pi l}\right)\sum_{q=0}^{N_3-1}\tilde{F}_2\left(\frac{z_q+z_{q+1}}{2}\right)e^{-i2\pi\frac{q}{N_3}l},\ \tilde{F}_3(z)=F_3(k+zl)\ \ z_q=\frac{q}{N_3}.$$

Theorem 6. For error $\left|CF_{k,l}-CF_{k,l,N}\right|$ of approach of Fourier's coefficients $CF_{k,l}$ of functions $f(x,y)\in C^r[0,1]^2$ by formulas $CF_{k,l,N}$ which turn out replacement of functions by wavelets W_{μ,N_μ} $\mu=1,2,3$ the estimation from above is fair (similar estimations also it is possible to write for $G_\mu,\phi_\mu,\omega_\mu$)

$$\left|CF_{k,l}-CF_{k,l,N}\right|=O\left(\left\|F_\mu(\cdot)-W_{\mu,N_\mu}(\cdot)\right\|_{C[0,1]}\right).$$

In particular, at approach by Haar's wavelets it is had $\left|CF_{k,l}-CF_{k,l,N}\right|=O\left(N^{-1}\right)$.

The analysis of results of computing experiment, in which for visualization of function $f(x,y)$ Fourier's and Fejer's were used, given in (Lytvyn, Pershina, Nechuyviter, Litvin, Kulyk, 2009).

Thus, by means of the received factors, it is possible to construct corresponding tomograms in a kind 2D Fourier sums.

4 METHOD OF CALCULATION OF FOURIER'S COEFFICIENTS FOR SECTIONS $f(x,y,z_\alpha),\ \alpha=\overline{0,M}$ BY MEANS OF THE ALGORITHM, USING OPERATORS INTERLINEATION WITH THE SET PROJECTIONS TO THE FIXED SYSTEM OF LINES.

Let the investigated object is located in square $D=E^3,\ E=[0,1]$ and its characteristic is defined by function $f(x,y,z)$,

$$f(x,y,z)=0,\ (x,y,z)\notin D.$$

Let's find Fourier coefficients for functions $f(x,y,z_s)$ as follows.

Let's spend through area $G=[0,1]^2$ straight lines $\Gamma_k: x=x_k(t),\ y=y_k(t),\ k=\overline{1,N}.$ It is considered known projections along these straight lines $\gamma_k=\gamma_{k,s}=\int\limits_{\Gamma_k} f(x,y,z_s)ds,\ k=\overline{1,N}$. We will state a method of restoration of function $f(x,y,z_s)$ with the help of projections along the specified lines on way.

For $z=z_s$ we build the interlineation operator $O_N(\{f_{k,s}\};U^s;x,y)=O_N(x,y,z_s)$ with properties

$$O_N(\{f_{k,s}\};U^s;x,y)=f(x,y,z_s)=f(x_k(t),y_k(t),z_s)=f_{k,s}(t),(x,y)\in\Gamma_k,\ k=\overline{1,N},$$

where t- parameter , $U^s=\left[U_{ij}^s\right]_{i=\overline{1,r}}^{j=\overline{1,p}}$ – unknown matrix.

Theorem 7. There are such operators $J_{k,s}=J_k(\gamma_{k,s};U^s;t)\approx f_{k,s}(t),\ k=\overline{1,N}$ that operator $O_N(\{J_{k,s}\};U^s;x,y)$ satisfies to conditions $\int\limits_{\Gamma_p} O_N(\{J_{k,s}\};U^s;x,y)ds=\gamma_p=\gamma_{p,s},\ s,p=\overline{1,N}$ irrespective of a choice of elements of matrix U^s..

Example. Let's

$$f(x,y,z_s)=f_s(x,y),\ N=m+n,$$

$$\gamma_{i,s}^{(2)}=\int_0^1 f_s(x_i,y)dy,\ i=\overline{1,m};\ \gamma_{j,s}^{(1)}=\int_0^1 f_s(x,y_j)dx,\ j=\overline{1,n}\ \text{– set numbers,}$$

$$0<x_i,y_j<1,\ i=\overline{1,m},\ j=\overline{1,n},\ x_0=y_0=0,\ x_{m+1}=y_{n+1}=1.$$

Let's

$$O_N(x,y,z_s)=O_N(\{\gamma_{i,s}^{(1)};\gamma_{j,s}^{(2)}\};U^s;x,y),$$

$$O_N(x,y,z_s)=\sum_{i=1}^{m}h_i(x)\left[\gamma_{i,s}^{(2)}+\sum_{j=1}^{n}\left(U_{ij}^s-\gamma_{i,s}^{(2)}\right)\varphi_j(y)\right]+$$

$$+\sum_{j=1}^{n}H_j(y)\left[\gamma_{j,s}^{(1)}+\sum_{i=1}^{m}\left(U_{ij}^s-\gamma_{j,s}^{(1)}\right)\psi_i(x)\right]-\sum_{i=1}^{m}\sum_{j=1}^{n}h_i(x)H_j(y)U_{ij}^s,$$

where functions $\psi_i(x)$, $\varphi_j(y)$, h_i, H_j have following properties

$$\varphi_j(y_q)=\delta_{j,q},\ \psi_i(x_p)=\delta_{i,p},\ i,p=\overline{0,m};\ \int_0^1\psi_i(x)dx=\int_0^1\varphi_j(y)dy=0;\ j,q=\overline{0,n},$$

$$h_i(x_p)=\delta_{i,p},\ i,p=\overline{0,m};\quad H_j(y_q)=\delta_{j,q},\ j,q=\overline{0,n}.$$

Theorem 8. For $O_N(x,y,z_s)$ next equation is valid

$$O_N(x_k,y_\ell,z_s)=U_{k\ell}^s\ \forall U_{k\ell}^s\in R,$$

$$\int_0^1 O_N(x_k,y,z_s)dy=\gamma_{k,s}^{(2)}=\int_0^1 f_s(x_k,y)dy,\ \int_0^1 O_N(x,y_\ell,z_s)dx=\gamma_{\ell,s}^{(1)}=\int_0^1 f_s(x,y_\ell)dx,\ k=\overline{1,m},\ \ell=\overline{1,n}.$$

Unknown $U_{k\ell}^s$ it is found from condition $R_1(U^s)\to\min\limits_{U^s}$, where

$$R_1(U^s)=\iint_D\left\{\lambda_0\left(O_N(\{\gamma_{j,s}^{(1)},\gamma_{i,s}^{(2)}\};U^s;x,y)\right)^2+\lambda_1\left[\left(\frac{\partial}{\partial x}O_N(\{\gamma_{j,s}^{(1)},\gamma_{i,s}^{(2)}\};\ U^s;x,y)\right)^2+\right.\right.$$

$$\left.\left(\frac{\partial}{\partial y}O_N(\{\gamma_{j,s}^{(1)},\gamma_{i,s}^{(2)}\};U^s;x,y)\right)^2\right]+$$

$$+\lambda_2\left[\left(\frac{\partial^2}{\partial x^2}O_N(\{\gamma_{j,s}^{(1)},\gamma_{i,s}^{(2)}\};U^s;x,y)\right)^2+2\left(\frac{\partial^2}{\partial x\partial y}O_N(\{\gamma_{j,s}^{(1)},\gamma_{i,s}^{(2)}\};U^s;x,y)\right)^2+\right.$$

$$\left.\left.+\left(\frac{\partial^2}{\partial y^2}O_N(\{\gamma_{j,s}^{(1)},\gamma_{i,s}^{(2)}\};U^s;x,y)\right)^2\right\}dxdy+\beta\left\|U^s\right\|^2,\quad \left\|U^s\right\|^2=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(U_{ij}^s\right)^2,$$

$\lambda_0,\lambda_1,\lambda_2,\beta$ – some numbers (β – regularization parameter). For not differentiated functions $f_s(x,y)$ it is possible to put $\lambda_0=1$, $\lambda_1=\lambda_2=0$. As a result for a finding of optimum point U^s we receive system of the linear algebraic equations

$$\frac{\partial R_1(U^s)}{\partial U_{p,q}^s}=0,\ p=\overline{1,m};\ q=\overline{1,n}.$$

After the decision this system of the linear algebraic equations for $z=z_s$, $s=\overline{1,M}$ we find $O_N(\{J_{k,s}\};U^s;x,y)=O_N(x,y,z_s)$ and can write for $f(x,y,z)$ next approximate 3D explicit form $f(x,y,z)\approx f^*(x,y,z)=\sum_{\alpha=1}^{M}O_N(x,y,z_s)h_s(z)$ and Fourier's coefficients of function $f^*(x,y,z)$ usual way, if it is necessary.

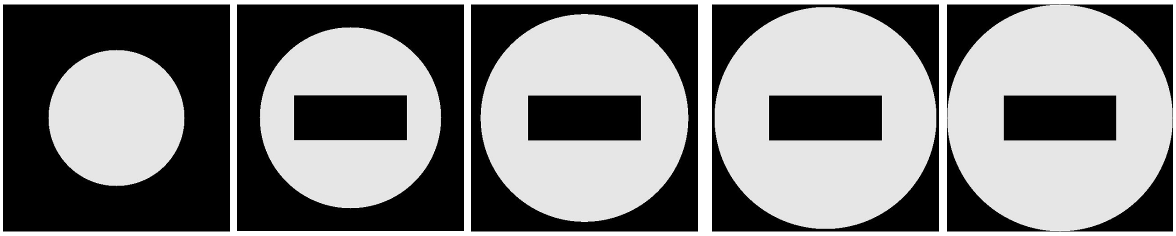
5 EXAMPLES

Example 1. Research of cracks in a sphere of radius 1 with the centre in a point (0.5,0.5,0.5) (for simplification we assume, that the crack looks like a parallelepiped). In this case investigated function looked like

$$f_1(x,y,z)=\begin{cases}1, \text{ if } (x-0.5)^2+(y-0.5)^2+(z-0.5)^2 \le 0.5^2 \wedge (|x-0.5| \ge a \vee |y-0.5| \ge b \vee |z-0.5| \ge c),\\ 0, \text{ if } (x-0.5)^2+(y-0.5)^2+(z-0.5)^2 > 0.5^2 \vee (|x-0.5| < a \wedge |y-0.5| < b \wedge |z-0.5| < c).\end{cases}$$

For the image of the first approached function its two-dimensional sections received by means of tomograms $f_1(x,y,z_k),\ z_k=\frac{k}{10},\ k=\overline{1,9}$ were used.

In fig. 1 we give only five sections as at $z>0.5$ images of sections will repeat, owing to symmetry of object with respect to plane $z=0.5$.



a) b) c) d) f)

Fig 1. The image of functions $f_1(x,y,z_k)$:

a) $f_1(x,y,0.1)$; b) $f_1(x,y,0.2)$, c) $f_1(x,y,0.3)$, d) $f_1(x,y,0.4)$; f) $f_1(x,y,0.5)$.

Example 2. Finding of Fourier's coefficients of the body having the form of "screw-driver" (a detail transferring twisting moment). In this case investigated function looked like

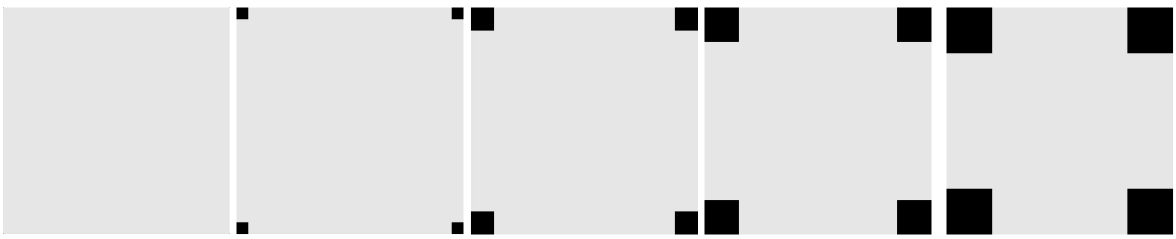
$$f_2(x,y,z)=Rd\left[w_1pm(x,y,z),w_2pm(x,y,z)\right],$$

where

$$w_1pm(x,y,z)=Rk\left[w_1p(x,z),w_1m(x,z)\right],\ w_2pm(x,y,z)=Rk\left[w_2p(y,z),w_2m(y,z)\right],$$
$$w_1p(x,z)=-(x+c\cdot z-d),\ w_1m(x,z)=-(-x+c\cdot z-d),\ w_2p(y,z)=-(y+c\cdot z-d),$$
$$w_2m(y,z)=-(-y+c\cdot z-d),\ Rd(u,v)=\frac{1}{2}\cdot(u+v+|u-v|),\ Rk(u,v)=\frac{1}{2}\cdot(u+v-|u-v|).$$

Sections of this body are set of planes $z=z_k,\ z_k=\frac{k}{10},\ k=\overline{0,9}$.

On fig. 2 images of functions $f_2(x,y,z_k)$ are presented. On fig. 2 are presented the level-by-level image of function $f_2(x,y,z_k)$ in the field of its definition



a) b) c) d) f)

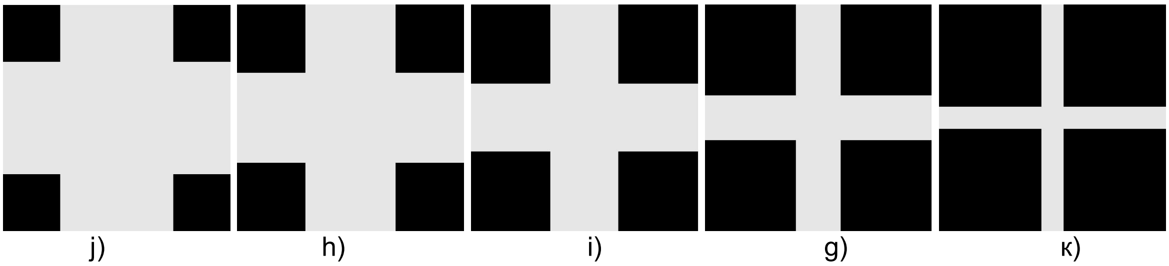


Fig. 2. The image of functions $f_2(x,y,z_k)$:

a) $f_2(x,y,0)$; b) $f_2(x,y,0.1)$, c) $f_2(x,y,0.2)$, d) $f_2(x,y,0.3)$; f) $f_2(x,y,0.4)$;

j) $f_2(x,y,0.5)$; h) $f_2(x,y,0.6)$; i) $f_2(x,y,0.7)$ g) $f_2(x,y,0.8)$; к) $f_2(x,y,0.9)$.

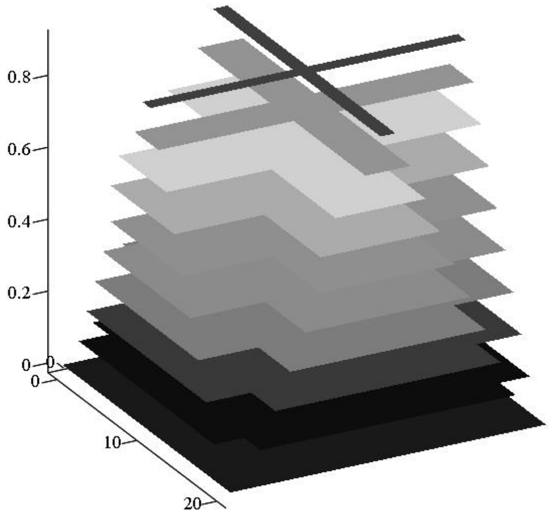


Fig.3. The level-by-level image of border of area in which $f_2(x,y,z)$ function is defined

Let's notice, that at construction of this function were used V. L. Rvachov's R – functions (Rvachev, 1982): R -conjunction – $Rk(u,v)$ and R -disjunction – $Rk(u,v)$.

6 REFERENCES

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