CONSTRUCTION METHODS OF 4D MATHEMATICAL MODELS 3D BODIES ON A BASIS INTERLINEATION, INTERFLATATION, BLENDING APPROXIMATION AND WAVELETS

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Abstract

In work the new method of the description of changes which occur in object changing in time, on the basis of use of the tomograms received during the various moments of time on system of planes which cross object is offered. Systems parallel or mutually perpendicular, or even crossed not under right angles of planes are supposed. The analysis of methods of construction of the tomograms based on use wavelets, interlineation functions of two variables, and approximation by Fourie and Fejer sums are given. During each moment of time we build 3D mathematical model on the basis of the constructed tomograms with the help: interflatation, the blending approximation, using of two or three x-ray pictures. On the basis of constructed 3D models during the various moments of time the four-dimensional mathematical model of the three-dimensional body changing in time is construction. Examples are considered.

1. Introduction

The problem of restoration of internal structure of the three-dimensional body changing in time, is one of the most actual problems of the present. It arises in various areas of a science and techniques. In particular, such problem arises in medical practice in case of carrying out of several repeated researches of the patient during the various moments of time and necessity of the analysis on their basis of efficiency of treatment, and also at processing of seismic signals at research of a bark of the Earth (Burmin 2008).

Construction 4D models on a basis tomographic data in the various moments of time is especially important. This problem in itself is very difficult in connection with the big volumes of the used information at each stage of time, and also in connection with obvious restrictions on visualisation of results of reconstruction in the moments of time which are not coinciding with the moments for which experimental data are given. Let's notice, that function of four variables can be visualised only on the values in separate points or the plot of the traces on separate lines or surfaces (in particular, planes).

In some cases (for example, at research 3D models of moving heart) are necessary for considering, that full change of internal structure of object is carried out approximately for one second. Therefore, if we wish to receive sequence of mathematical models of a three-dimensional body during the various moments of time for each of these moments of time it is necessary to perform great volume of work. For research of changes on time and, in particular, for the prognosis, obviously, it is necessary analytical on t, x, y, z representation of internal structure of a three-dimensional body.

Considering stated, the problem of construction analytical 4D models of changing object on the basis of tomograms during the various moments of time is actual.

2. Statement of the problem

Let's designate through $f(x, y, z, t)$ exact analytical representation of some characteristic of a body (for example, density or absorption factor). As experimental data we will use:

1. Sequence of the moments of time: $t_1 < t_2 < ... < t_n$;

1. Series s of the planes set by the equation: $\Pi_p : \omega_p(x, y, z) = a_{p1}x + a_{p2}y + a_{p3}z - \gamma_p = 0,\ \ p = \overline{1,s}$, on which there are the tomograms representing functions-matrixes $\varphi_{k,p}(x, y)$ or $\varphi_{k,p}(x, z)$ or $\varphi_{k,p}(y, z)$.

It is required under given information to construct function $F(x, y, z, t)$ with properties:

$$F(x, y, z, t) \in C(D),\ \ f_k(x, y, z) = f(x, y, z, t_k),\ k = \overline{1,n}$$
$$F(x, y, z, t_k)\big|_{\omega_p(x,y,z)=0} = f_k\big|_{\omega_p(x,y,z)=0} = \varphi_{kp}(u, v),\ k = \overline{1,n},\ \ p = \overline{1,s}.$$

The problem in such statement has no unique decision. However, at certain restrictions on a class of approximated functions it will have the unique decision. Moreover, for some classes of approached functions $f(x, y, z, t)$ the approach error can be estimated.

For the task in view decision in the given work it is offered to build during each moment of time $t = t_k$, $k = \overline{1,n}$ corresponding mathematical model of internal structure of a three-dimensional body in the form of function $f_k(x, y, z)$ which has the following property:

$$f_k(x, y, z)\big|_{\Pi_p} = \varphi_{k,p}(u, v),\ (u, v) \in \Pi_p, k = \overline{1,n},\ p = \overline{1,s}.$$

That is function $f_k(x, y, z)$ on plane Π_p coincides with image p - th tomograms. For construction of these functions can be used operators a spline-interflatation, and also operators of the blending approximation (Lytvyn, 2006).

After that 4D the mathematical model is under construction in the form of corresponding formulas of interpolation or approximation on t in a kind

$$F(x, y, z, t) = \sum_{k=1}^{n} h_k(t) f_k(x, y, z)\text{, where } h_k(t) \text{ – auxiliary functions.}$$

In work as an example the following problem solves: to construct 4D mathematical model of heart changing in time on the basis of the given (tomograms) received by means of magnatic-resonance computer tomograph Siemens of the kind permission of madam Tatyana A.Jalinskoj, managing radiological branch of the cardiological centre at institute of protection of mother and the child (Kiev, Ukraine).

3. The basic definitions

Let are given function of four variables $f(x,y,z,t)$ in point (x,y,z), changing in time t, and system of planes $\Pi_p : \omega_p(x,y,z) = a_{p1}x + a_{p2}y + a_{p3}z - \gamma_p = 0,\ p = \overline{1,s}$.

Definition 1. After function $f(x,y,z,t_k)$ at the moment of time t_k, $k = \overline{1,n}$ for planes Π_p we will name function of two variables $\varphi_{k,p}(x,y)$ or $\varphi_{k,p}(x,z)$ or $\varphi_{k,p}(y,z)$, with properties

$$f_k(x,y,z)\big|_{\Pi_p} = \varphi_{k,p}(u,v),\ (u,v) \in \Pi_p,\ k = \overline{1,n},\ p = \overline{1,s},\ (u,v) \in \{(x,y),(x,z),(y,z)\} \tag{1}$$

Definition 2. Interflatation of functions $f(x,y,z,t)$ is restoration (probably, approached) functions $f(x,y,z,t_k)$ in points between planes Π_p by means of its traces (1) on these planes at the moment of time t_k is called.

Definition *3.* Tomogram $T_{k,p}(\bar{x})$ (after function $f(x,y,z,t_k)$) on plane $\omega_p(x,y,z) = 0$ at the moment of time t_k, $k = \overrightarrow{1,n}$ we will name one of three functions:

$$\mathrm{T}_{k,p}(\bar{x}) = \begin{cases} f(x_p(y,z),y,z,t_k) \\ f(x,y_p(x,z),z,t_k) \\ f(x,y,z_p(x,y),t_k) \end{cases};\ \bar{x} = \begin{cases} (x,y) \\ (x,z) \\ (y,z) \end{cases}$$

where $x_p(y,z),\ y_p(x,z),\ z_p(x,y)$ - decisions of equation $\omega_p(x,y,z) = 0$.

At the moment of time t_k which lays, for example, for planes $\omega_p(x,y,z) := z - c = 0$ it is possible to present the tomogram in the form of trace $f(x,y,c,t_k) = T_{kp}(x,y)$ (see figure 1)

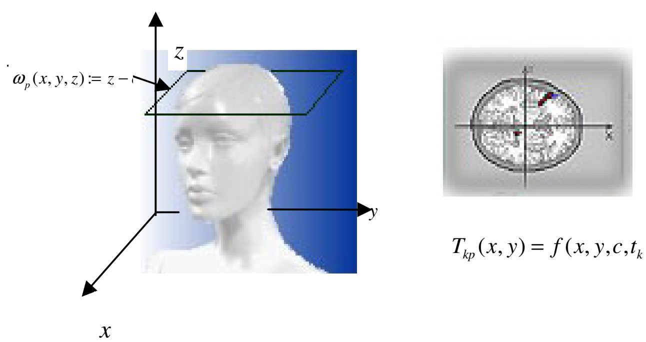


Figure 1: Representation of the tomogram in the form of function.

4. Construction method 4D mathematical model

Let are set n the moments of time $t_1 < t_2 < ... < t_n$, system of any planes Π_p and tomograms $T_{kp}, k = \overline{1,n},\ p = \overline{1,s}$ of three-dimensional object on the set planes and during the set moments of time. We receive tomograms by computer tomograph. I.e. we have n groups of tomograms (in each group on s tomograms), in each of which tomograms are presented to the same moment of time, but laying on different planes. At first we will construct n three-dimensional mathematical models of internal structure of object $f_k(x,y,z),\ k = \overline{1,n}$ for each of the set groups of tomograms. For construction of these models it is possible to use the high-precision methods developed by authors in works (Lytvyn, Pershina, 2005; Lytvyn, Lytvyn, Mezhuyev, Babenko, Pershina, 2007; Lytvyn, 2006; Lytvyn, Pershina, 2008):

– if experimental data (geometrical parametres of planes in which tomograms lay, and also the image on tomograms) are set precisely, it is possible to use a method of restoration internal structure of a three-dimensional body by interflatation functions (Oleg N. Lytvyn, I. Pershina. (2005).);

– if experimental data are have errors, it is possible to use a method of a three-dimensional computer tomography by blending approximation (Oleg M. Lytvyn, Yuliya I. Pershina (2008)).

The specified methods of restoration internal structure of a three-dimensional body have high accuracy. After set construction 3D models $f_k(x,y,z)$ we build 4D mathematical model $F(x,y,z,t)$, using an interpolation method on variable t by the formula:

$$F(x,y,z,t)=\sum_{k=1}^{n} h_k(t)\, f_k(x,y,z)\,,$$

where $h_p(t)$ - auxiliary functions from one variable t with properties:

$$h_k(t_q)=\delta_{kq}\,,\ \delta_{kq}=1,\ if\ \ k=q;\ \delta_{kq}=0,\ if\ \ k\neq q\ .$$

These functions can be splines of degree r, $r=1,2,3,\ldots$, polynoms of degree $n-1$ or trigonometrical polynoms.

5. The method of solution 2D problems of a x-ray computer tomography, based on explicit formulas for operators interlineation with the set projections

In this section the method of receiving of tomograms with use of operators interlineation is stated.

Let the investigated object is located in square $\mathrm{D}=\mathrm{E}^2,\ \mathrm{E}=[0,1]$ and its characteristic is defined by function $f(x,y)$, $f(x,y)=0,(x,y)\notin D$. Let $\Gamma_k : x=x_k(t), y=y_k(t), k=\overline{1,N}$ are straight lines and $\Gamma_k \bigcap D\neq\varnothing,\ k=\overline{1,N}$. Let projections $\gamma_k=\int\limits_{\Gamma_k} f(x,y)ds, k=\overline{1,N}$ are known. We will state a method. We build the operator of interlineation $O_N(\{f_k\};U;x,y)$ with properties

$$O_N(\{f_k\};U;x,y)=f(x,y)=f(x_k(t),y_k(t))=f_k(t),(x,y)\in\Gamma_k,\ k=\overline{1,N},$$

where t - parametre, $U=\left[u_{ij}\right]_{i=\overline{1,r}}^{j=\overline{1,p}}$ – unknown matrix.

Theorem. There are such operators $J_k=J_k(\gamma_k;U;t)\approx f_k(t), k=\overline{1,N},$ that operator $O_N(\{J_k\};U;x,y)$ satisfies to conditions $\int\limits_{\Gamma_p} O_N(\{J_k\};U;x,y)ds=\gamma_p,\ p=\overline{1,N}$ irrespective of a choice of elements of matrix U.

*Example***.** Let $N=m+n$, $\mathrm{x_i}, \gamma_i^{(2)}=\int_0^1 f(x_i,y)dy, i=\overline{1,m};\ \mathrm{y_j}, \gamma_j^{(1)}=\int_0^1 f(x,y_j)dx,\ j=\overline{1,n}$ - the set numbers, $0<x_i,y_j<1, i=\overline{1,m},\ j=\overline{1,n}, x_0=y_0=0, x_{m+1}=y_{n+1}=1$.

Let $O_{mn}(x,y) =: \mathrm{O_{mn}}(\{\gamma_{\mathrm{i}}^{(1)};\gamma_{\mathrm{j}}^{(2)}\};\mathrm{U;x,y})$, $O_{m,n}(x,y) = \sum_{i=1}^{m} h_i(x)\left[\gamma_i^{(2)} + \sum_{j=1}^{n}\left(U_{ij} - \gamma_i^{(2)}\right)\varphi_j(y)\right] +$

$$+\sum_{j=1}^{n} H_j(y)\left[\gamma_j^{(1)} + \sum_{i=1}^{m}\left(U_{ij} - \gamma_j^{(1)}\right)\psi_i(x)\right] - \sum_{i=1}^{m}\sum_{j=1}^{n} h_i(x) H_j(y) U_{ij},$$

where functions $\psi_i(x)$, $\varphi_j(y)$, h_i, H_j have following properties

$$\varphi_j(y_q) = \delta_{j,q},\ \psi_i(x_p) = \delta_{i,p}, i,p = \overline{0,m}; \quad \int_0^1 \psi_i(x)\,dx = \int_0^1 \varphi_j(y)\,dy = 0; j,q = \overline{0,n},$$

$$h_i(x_p) = \delta_{i,p}, i,p = \overline{0,m}; \quad , \quad H_j(y_q) = \delta_{j,q}, j,q = \overline{0,n}.$$

Then: $O_{mn}(x_k, y_\ell) = U_{k\ell} \forall U_{k\ell} \in R$,

$$\int_0^1 O_{MN}(x_k, y)dy = \gamma_k^{(2)} = \int_0^1 f(x_k, y)dy,\ \int_0^1 O_{MN}(x, y_\ell)dx = \gamma_\ell^{(1)} = \int_0^1 f(x, y_\ell)dx,\ k = \overline{1,m},\ \ell = \overline{1,n}$$

Unknown $U_{k,l}$ it is found from condition $R_1^\alpha(U) \to \min\limits_U$.

$$\begin{aligned}\Omega_1(U) = \iint_D \Big\{ &\lambda_0\left(O_{MN}(\{\gamma_j^{(1)},\gamma_i^{(2)}\};U;x,y)\right)^2 + \lambda_1\left[\left(\frac{\partial}{\partial x}O_{MN}(\{\gamma_j^{(1)},\gamma_i^{(2)}\};U;x,y)\right)^2 + \left(\frac{\partial}{\partial y}O_{MN}(\{\gamma_j^{(1)},\gamma_i^{(2)}\};U;x,y)\right)^2\right] + \\ &+\lambda_2\left[\left(\frac{\partial^2}{\partial x^2}O_{MN}(\{\gamma_j^{(1)},\gamma_i^{(2)}\};U;x,y)\right)^2 + 2\left(\frac{\partial^2}{\partial x\partial y}O_{MN}(\{\gamma_j^{(1)},\gamma_i^{(2)}\};U;x,y)\right)^2 + \right. \\ &\left. +\left(\frac{\partial^2}{\partial y^2}O_{MN}(\{\gamma_j^{(1)},\gamma_i^{(2)}\};U;x,y)\right)^2\right]\Big\} dxdy + \alpha\|U\|^2, \|U\|^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} U_{ij}^2.\end{aligned}$$

where λ_0, $\lambda_1, \lambda_2, \alpha$ - some numbers (α – parametre of regularization). For not differentiated functions $f(x,y)$ it is possible to put $\lambda_0 = 1$, $\lambda_1 = \lambda_2 = 0$.

As a result for a finding of optimum point U we receive system of the linear algebraic equations

$$\frac{\partial\Omega_1(U)}{\partial U_{p,q}} = 0,\ p = \overline{1,m}; q = \overline{1,n}$$

6. 3D models of internal structure of a body by 2 or 3 x-ray pictures in mutual perpendicular directions.

Let $f(x,y,z)$ - analytical expression for internal structure 3D bodies. Let's enter operators

$$c1(f;y,z) = \int_0^1 f(x,y,z)dx,\ c2(f;x,z) = \int_0^1 f(x,y,z)dy,\ c12(f;z) = \int_0^1\int_0^1 f(x,y,z)dxdy.$$

Formula $Bf(x,y,z) = c1(f;y,z) + c2(f;x,z) - c12(f;z)$ is mathematical model of internal structure 3D bodies wich uses 2 X-ray pictures in is mutual-perpendicular directions Ox, Oy and has properties

$$\int_0^1 Bf(x,y,z)\,dx=\int_0^1 f dx = c1(f;y,z),\qquad \int_0^1 Bf(x,y,z)\,dy=\int_0^1 f dy = c2(f;x,z)$$

Theorem. $B_{0,0}f = f\,\forall f(x,y,z)=u(x,z)+v(y,z)$, where $u(x,z), v(y,z)$ - any integrated functions of two variables.

Formula

$$Bf(x,y,z)=c1(f;y,z)+c2(f;x,z)+c3(f;x,y)-$$
$$-c1(c2(f;.,.);.,z)-c1(c3(f;.,y);y,.)-c2(c3(f;.,.);x,.)+c123$$

is mathematical model of internal structure 3D bodies which uses 3 x-ray pictures in mutually perpendicular directions Ox, Oy, Oz and

$$\int_0^1 B_{0,0,0}f\,dx=\int_0^1 f\,dx=c1(f;y,z);\ \int_0^1 B_{0,0,0}f\,dy=\int_0^1 f\,dy=c2(f;x,z);$$
$$\int_0^1 B_{0,0,0}f\,dz=\int_0^1 f\,dz=c3(f;x,y),\int_0^1\int_0^1\int_0^1 f dxdydz=c123.$$

Theorem. $Bf = f\,\forall f(x,y,z)=u(y,z)+v(x,z)+w(x,y)$, where $u(x,z), v(y,z), w(x,y)$ - any integrated functions.

7. Method of calculation of Fourier coefficients of two variables functions by finitary Haar sums

The new method of calculation of Fourier coefficients of two variables functions $f(x,y)$ which is used at mathematical modelling in a computer tomography by Fourier or Fejer trigonometrical polynoms is offered. The method uses replacement of projections (integrals from function $f(x,y)$ along the set of lines which cross object of research) the corresponding wavelet sums.

Let's use such auxiliary statements.

Lemma. At calculation of Fourier coefficients $CF_{k,l}=\int_0^1\int_0^1 f(x,y)e^{-i2\pi(kx+ly)}dxdy$ for *k = 0, l = 0; k = 0, l> 0; k> 0, l = 0* by projections equalities are fair

$$CF_{00}=\int_0^1\int_0^1 f(x,y)\,dxdy=\int_0^1\gamma_2(y)\,dy,\ \gamma_2(y)=\int_0^1 f(x,y)\,dx$$

$$CF_{k,0}=\int_0^1\int_0^1 f(x,y)e^{-i2\pi kx}dxdy=\int_0^1\gamma_1(x)e^{-i2\pi kx}dx,\ \gamma_1(x)=\int_0^1 f(x,y)\,dy$$

$$CF_{0,l}=\int_0^1\int_0^1 f(x,y)e^{-i2\pi ly}dxdy=\int_0^1\gamma_2(y)e^{-i2\pi ly}dy$$

For *k < 0, l < 0* we have $CF_{-|k|,0}=\overline{CF_{|k|,0}}$; $CF_{0,-|l|}=\overline{CF_{0,|l|}}$.

For $k\ge 1$, $l\ge 1$ and $k\ge l$: $CF_{k,l}=I_1+I_2+I_3$,

$$I_1=l\int_0^1\frac{F_1(zl)e^{-i2\pi zl}dz}{k^2+l^2},\ I_2=\frac{k-l}{k^2+l^2}\int_0^1 F_2(l+z(k-l))e^{-i2\pi[l+z(k-l)]}dz,$$

$$I_3 = l\int_0^1 \frac{F_3(k+zl)e^{-i2\pi(k+zl)}dz}{k^2+l^2}, \quad \text{where } F_1(t) = \int_{-\frac{l}{k}t}^{\frac{k}{l}t} f\left(\frac{kt-lv}{k^2+l^2}, \frac{lt+kv}{k^2+l^2}\right)dv,$$

$$F_2(t) = \int_{-\frac{l}{k}t}^{\frac{k^2+l^2-lt}{k}} f\left(\frac{kt-lv}{k^2+l^2}, \frac{lt+kv}{k^2+l^2}\right)dv, \; F_3(t) = \int_{\frac{1}{l}\left[-k^2-l^2+kt\right]}^{\frac{1}{k}\left[k^2+l^2-lt\right]} f\left(\frac{kt-lv}{k^2+l^2}, \frac{lt+kv}{k^2+l^2}\right)dv.$$

Similar formulas for other values of indexes k and l use other designations for projections (instead of F_μ designations G_μ, ϕ_μ, ω_μ, $\mu = 1,2,3$). We omit them.

Let's notice, that for values k, l, satisfying to conditions $k,\ l \le -1$, equalities $CF_{-k,-l} = \overline{CF_{k,l}}$; $CF_{-k,l} = \overline{CF_{k,-l}}$, $\overline{\alpha + i\beta} := \alpha - i\beta$ are used. Written above function F_μ, G_μ, ϕ_μ, ω_μ, $\mu = 1,2,3$ are the projections received by integration of function $f(x,y)$ along straight lines, crossing square $[0,1]^2$ and pass in parallel direct $kx + ly = t$. In practice the specified experimental data can be received by computer tomograph for a discrete set of values of variable t.

The method consists in replacement of functions $\gamma_1(x)$, $\gamma_2(y)$, $F_1(zl)$, $F_2(l + z(k-l))$, by Haar wavelet sums and in exact calculation of the received integrals. It is possible to use except Haar wavelets as well others wavelets.

Theorem. For the approached calculation of Fourier coefficients by finite Haar sum wavelets and a discrete set of projections we have formulas

$$CF_{0,0} \approx CF_{0,0,M} = \frac{1}{M}\sum_{p=1}^{M}\gamma 1_p = \frac{1}{M}\sum_{q=1}^{M}\gamma 2_q, \; CF_{k,0} \approx CF_{k,0,M} = \sum_{p=1}^{M}\gamma 1_p e^{-i\frac{p}{M}},$$

$$CF_{0,l} \approx CF_{0,l,M} = \sum_{q=1}^{M}\gamma 2_q e^{-i\frac{q}{M}},$$

$$CF_{k,l} \approx CF_{k,l,N} = I_{1,k,l,N_1} + I_{2,k,l,N_2} + I_{3,k,l,N_3}, \quad k \ge l \ge 1,$$

$$I_{1,k,l,N_1} = \frac{l}{k^2+l^2}\left(\frac{e^{-i2\pi\frac{l}{N_1}} - 1}{-i2\pi l}\right)\sum_{q=0}^{N_1-1}\tilde{F}_1\left(\frac{z_q + z_{q+1}}{2}\right)e^{-i2\pi\frac{q}{N_1}l}, \; \tilde{F}_1(z) = F_1(zl), \; z_q = \frac{q}{N_1},$$

$$I_{2,k,l,N_2} = \frac{k-l}{k^2+l^2}\left(\frac{e^{-i2\pi\frac{k-l}{N_2}} - 1}{-i2\pi(k-l)}\right)\sum_{q=0}^{N_2-1}\tilde{F}_2\left(\frac{z_q + z_{q+1}}{2}\right)e^{-i2\pi\frac{q}{N_2}(k-l)}, \; \tilde{F}_2(z) = F_2(l + z(k-l)),$$

$$z_q = \frac{q}{N_2},$$

$$I_{3,k,l,N_3} = \frac{l}{k^2+l^2}\left(\frac{e^{-i2\pi\frac{l}{N_3}} - 1}{-i2\pi l}\right)\sum_{q=0}^{N_3-1}\tilde{F}_2\left(\frac{z_q + z_{q+1}}{2}\right)e^{-i2\pi\frac{q}{N_3}l}, \; \tilde{F}_3(z) = F_3(k + zl) \;\; z_q = \frac{q}{N_3}.$$

Theorem. For error $\left|CF_{k,l}-CF_{k,l,N}\right|$ of approach of Fourier coefficients $CF_{k,l}$ of functions $f(x,y)\in C^r[0,1]^2$ by formulas $CF_{k,l,N}$ which turn out replacement of functions by wavelets W_{μ,N_μ} $\mu=1,2,3$ the estimation from above is fair (similar estimations also it is possible to write for $G_\mu,\phi_\mu,\omega_\mu$)

$$\left|CF_{k,l}-CF_{k,l,N}\right|=O\left(\left\|F_\mu(\cdot)-W_{\mu,N_\mu}(\cdot)\right\|_{C[0,1]}\right).$$

In particular, at approach by Haar wavelets it is had $\left|CF_{k,l}-CF_{k,l,N}\right|=O\left(N^{-1}\right)$.

Let's bring analysis of results of computing experiment, in which for visualization of function $f(x,y)$ Fourier and Fejer sums were used

$$SF(x,y,N)=\sum_{k=-N}^{+N}\sum_{\ell=-N}^{+N}CF_{k,\ell}e^{i2\pi(kx+\ell y)},$$

$$SFE(x,y,N)=\sum_{k=-N}^{+N}\sum_{\ell=-N}^{+N}\left(1-\frac{|k|}{N+1}\right)\left(1-\frac{|l|}{N+1}\right)CF_{k,\ell}e^{i2\pi(kx+\ell y)}.$$

For smooth functions computing experiment has confirmed theoretical statements article. Therefore, call interest of possibility of the offered method for restoration of piece-wise-smooth functions. In the examples brought more low it is restored by given projections coefficient of absorption which is defined by explosive functions

$$f_1(x,y)=\begin{cases}1, & (x-0,5)^2+(y-0,5)^2\le r^2,\\ 0, & otherwise,\end{cases}\qquad f_2(x,y)=\begin{cases}1, & w(x,y)>0,\\ 0, & otherwise.\end{cases}$$

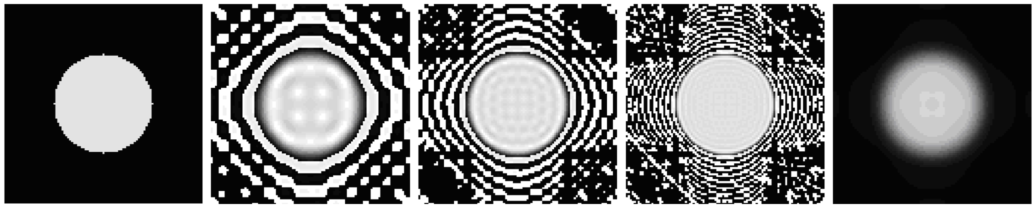
where $w=u+v+\sqrt{u^2+v^2}$, $u=-(x-0,5-a)(x-0,5+a)$, $v=-(x-0,5-b)(x-0,5+b)$, $a=b=r=0,25$.

In table 1 are brought maximum on the module of an error of approach of Fourier coefficients for $N=8,\ N=16,\ N=32$.

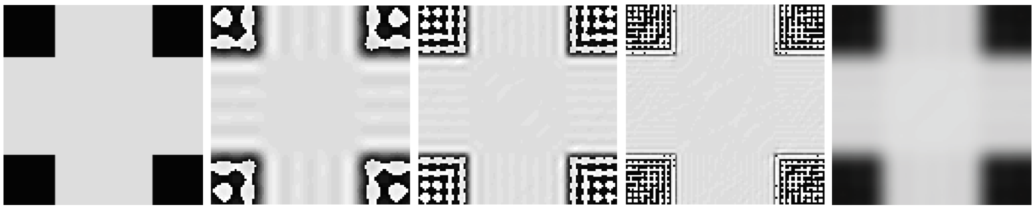
Table 1.

Function	N	$\varepsilon=\max\limits_{1\le k,l\le N}\left\|C_{k,l}-\tilde{C}_{k,l}\right\|$	Function	N	$\varepsilon=\max\limits_{1\le k,l\le N}\left\|C_{k,l}-\tilde{C}_{k,l}\right\|$
$f_1(x,y)$	8	$4,736\times10^{-4}$	$f_2(x,y)$	8	$9,166\times10^{-4}$
	16	$4,968\times10^{-4}$		16	$9,23\times10^{-4}$
	32	$4,968\times10^{-4}$		32	$9,23\times10^{-4}$

Schedules of these functions of two variables $f(x,y)$ and their Fourier and Fejer sums were under construction in shades grey as it is accepted in a computer tomography. We will bring, as an example, visualisation of results of restoration of functions $f_i(x,y)$, $i=1,2$



a) b) c) d) e)



a) b) c) d) e)

Figure 2*: Approach of functions $f_i(x, y)$, i=1,2 the offered method:*

a) – image of function $f(x, y)$; b), c), d) – image of Fourier sums for N = 8, N = 16, N = 32; e) – image of Fejer sums for N = 8

8. Computing experiment

Authors had been made computing experiment for restoration of internal structure of heart of the person changing in time.

As experimental data were are taken

1. $n = 25$ moment of time; 2. $s = 9$ parallel sections of heart by planes, perpendicular axes Ox (the method allows to use along with the set tomograms also the tomograms laying in other sections, not perpendicular axes Ox); 3. Tomograms of the heart, laying on nine set planes in each of 25 moments of time.

That is we have 25 groups of tomograms. In each group are presented tomogramms, made in one certain moment of time, in nine sections (i.e in each group on 9 tomograms). On fig. 3-4 examples of tomograms during the different moments of time in the same sections are shown.

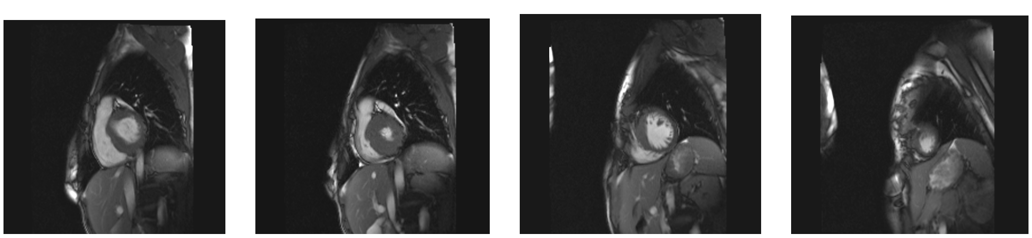


Figure 3. Examples of the tomograms made at the moment of time t_1, in the section $x = 1,\ x = 3,\ x = 5,\ x = 8$

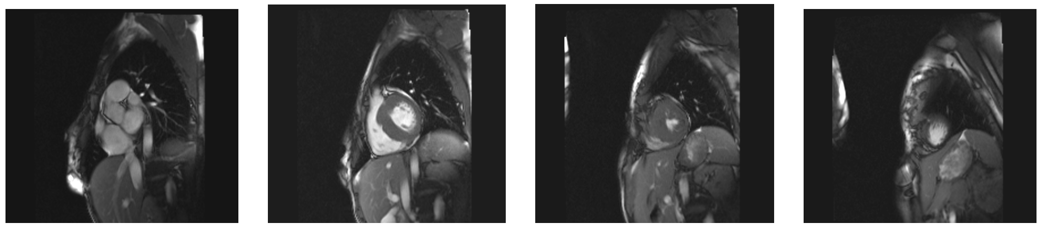


Figure 4. Examples of the tomograms made at the moment of time t_7 ,

in the section $x=1,\ x=3,\ x=5,\ x=8$

On the basis of a method developed in work (Lytvyn, Pershina, 2005), we had been constructed 25 mathematical models of internal structure in the form of functions $f_k(x,y,z),\ k=\overline{1,25}$. Then these mathematical models have been used at construction 4D mathematical model by a method stated in work (Lytvyn, Pershina 2008).

By means of constructed 4D mathematical model we will find the image of heart of the person in a plane set by equation $x=a$, during the concrete moment of time $t=t_{prognosis}$.

On fig. 5 results of computing experiment are present.

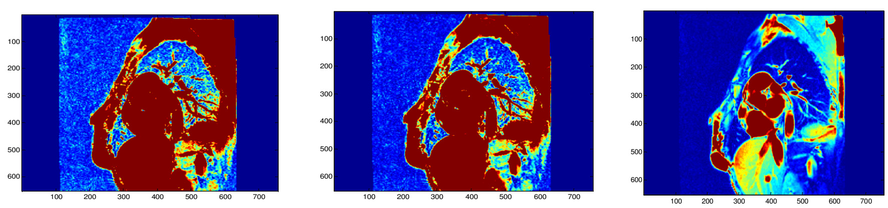


Figure. 6: Restoration of internal structure of heart in plane $x=0.1$ during the moments of time $t_5,\ t_{12},\ t_{20}$ accordingly.

Similar examples can be brought for construction 4D mathematical model of the body constructed on a basis 2 or 3 x-ray pictures in mutually perpendicular directions by blending approximation of three variables functions (see section 6), and also by interlineations functions (see section 5) and by wavelet-approximation (see section 7). The given technique can be used for testing of aircraft engines and for the analysis of processes in silage towers (K. GRUDZIEŃ, Z. CHANIECKI, M. NIEDOSTATKIEWICZ, A. ROMANOWSKI, D. SANKOWSKI (2008)).

9. Conclusion

Thus, in the given article the method of construction of operators with the set projections on lines of interloneation, crossing object of research is stated. In these operators obviously set projections, and also unknown parametres which are from a condition of a minimum corresponding functional enter. I.e. the offered method can be considered as generalisation of an algebraic method. The example is brought.

For the decision 3D problems are offered formulas for mathematical models of internal structure of the bodies, using 2 or 3 x-ray pictures in mutual-perpendicular directions. Classes of the objects which internal structure is precisely restored by these operators are given.

These mathematical models can be effectively used at creation of mathematical models of a body in which the internal structure of a body changes in time.

10. References

V.Y. BURMIN (2008). Viscosity of a terrestrial kernel on seismic data, *Dokladi RAN*, Vol. 418, № 3,4,5,6, pp. 825-828, Russa

OLEG N. LYTVYN, YULIA I. PERSHINA. (2005). Reconstruction of 3 – D objects with use interflation of functions. *Signal and image processing: Proceeding of the Second IASTED International Multi – Conference on Automation, Control, and Information Technology (June 20 – 24 2005).* – Novosibirsk. pp.274 – 279, Russa

O.N. LYTVYN, O.O. LYTVYN, V.I. MEZHUYEV, K.E. BABENKO, Y.I.PERSHINA (2007). Operators of the interflatation of functions of 3 variables in the 3D computer tomography. *Proceedings of the 5th World Congress on Industrial Process Tomography (3–6 September 2007)).* – Bergen, pp. 242 – 249, Norway

OLEG N. LYTVYN (2006). Interlineation and interflatation functions of many variable (blending function interpolation) and economical algorithms in the approximation theory. *In Book. Computational methods.* Part 2. (G.R.Liu, V.B.C. Tan, X Han-editors). pp. 1105 – 1110.

OLEG M. LYTVYN, YULIYA I. PERSHINA (2008) A method for 3D computer tomography using blending approximation. *Proceedings of the 5th International Symposium on Process Tomography (25–26 August 2008).* – Zakopane, 2008. PT08_cr9, Poland

OLEG M. LYTVYN, OLEG O. LYTVYN (1998). Viscosity of a terrestrial kernel on seismic data, *Dokladi RAN*, Vol. 418, № 3,4,5,6, pp. 825-828, Russa

IVAN V. SERGIENKO, OLEG O. LYTVYN (2008). Mathematical modelling of internal structure 3D bodies on the basis of three x-ray pictures in three is mutual-perpendicular foreshortenings.–Dopovidi NAN Ukraine, №8. – pp. 29-34

IVAN V. SERGIENKO, OLEG O. LYTVYN (2008). Mathematical modelling of internal structure 3D bodies on the basis of two x-ray pictures in three is mutual-perpendicular foreshortenings.–Dopovidi NAN Ukraine, №7. – pp. 23-28

K. GRUDZIEŃ, Z. CHANIECKI, M. NIEDOSTATKIEWICZ, A. ROMANOWSKI, D. SANKOWSKI (2008) Description Of The Dynamic Silo Discharging In Tomographical Process Diagnosis. *Proceedings of the 5th International Symposium on Process Tomography (25–26 August 2008).* – Zakopane, 2008. PT08_cr27, Poland