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THE STOCHASTIC ANALYSIS OF THE DISTRIBUTION OF CONTROL INFORMATION FOR DIRECT NUMERICAL CONTROL SYSTEMS

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The queueing model of direct numerical control systems which describes the process of distribution of control information is studied. Basic performance quantities in dependence on system parameters are obtained.

1. Utilization of direct numerical control systems (DNC) is important for the automation of production. The special machine tool control programs (MCP) are used for the operation of numerically controlled machine tools. These programs are stored in computer memory and presented as single characters of MCP frames via interface to the numerically controlled controllers of machine tools on their requests. A malfunction in the operation of machine tools is if the computer will not transmit the next frame of control information during service of the current request. It is possible, taking into account the time quantities of the interface between the computer and controllers of machine tools, to implement simultaneous distribution of MCP frames to the batch of machine tools, which have dispatched requests, in time sharing manner. The rate of frames transmission μ_j may be dependent on the number of simultaneously transmitted frames j in batch. A new arrival request would have been waiting untill release of the computer if it is busy with distribution of MCP frames. A new batch of requests would service after release of the computer. Number of machine tools in the system n is finite. Taking into account the Markovian nature of queueing processes in DNC systems [1, 2] we would use a single server Poisson queueing system with finite number of request sources as model of DNC systems. In this model unlike the same, studied in [3], the choice of requests on service is accomplished by batch on some size and rate of service is dependent on the number of requests in batch.

2. Let us define n as the number of request sources in the system (this number is equal to the number of machine tools in the DNC system); choice of requests on service is accomplished by batch on size j $(1 \leqq j \leqq m \leqq n)$, where m is the highest number of simultaneously serviced requests, mean arrival rate of requests from one source is λ, service rate μ_j is the function of the number of requests in batch ($\mu_j = \mu_1 v(j)$, where μ_1 is the rate of service, when the number of requests in batch is equal to one).

Let $\{i,j\}$ denote the state of the system, when j requests are on service, i sources do not send requests and $n-i-j$ requests are waiting for service. In the beginning of operation $j=0$, $i=n$; if $n-i-j>m$ then after service of current batch m requests are chosen from the queue and j would be equal to m (it would be $i+j$ requests in the source and $n-i-j-m$ requests in the queue); if $n-i-j\leq m$, then after service of current batch all requests from the queue would be chosen for service and j would be equal to $n-i-j$. Let $p_{i,j}$ be the stationary probability of the state $\{i,j\}$. Denote the ratio $\mu_{j/\lambda}$ as ψ_j, then taking into account the Markovian nature of process in system and examining all transitions among states we can deduce such a set of equations for probabilities $p_{i,j}$

$$
\begin{aligned}
&-np_{n,0}+\sum_{j=1}^{m}\psi_j p_{n-j,j}=0;\\
&-(n-1+\psi_1)\,p_{n-1,1}+\sum_{j=1}^{m}\psi_j p_{n-1-j,j}+np_{n,0}=0;\\
&-(j+\psi_1)\,p_{j,1}+(j+1)\,p_{j+1,1}=0, \qquad j=\overline{0,n-2};\\
&-(n-i+\psi_i)\,p_{n-i,i}+\sum_{j=1}^{\min(n-i,m)}\psi_j p_{n-i-j,j}=0, \qquad i=\overline{2,m-1};\\
&-(j+\psi_i)\,p_{j,i}+(j+1)\,p_{j+1,i}=0, \qquad i=\overline{2,m-1}, \qquad j=\overline{0,n-i-1}; \qquad\qquad (1)\\
&-(n-m+\psi_m)\,p_{n-m,m}+\sum_{j=1}^{\min(n-m,m)}\psi_j p_{n-m-j,j}=0;\\
&-(n-m-i+\psi_m)\,p_{n-m-i,m}+(n-m-i+1)\,p_{n-m-i+1,m}+\\
&\quad+\sum_{j=1}^{\min(n-m-i,m)}\psi_j p_{n-m-j-i,j}=0, \qquad i=\overline{1,n-m-1};\\
&-\psi_m p_{0,m}+p_{1,m}=0.
\end{aligned}
$$

The number of equations in this set is equal to $0.5\,m\,(2n+1-m)+1$. The normalizing condition

$$p_{n,0}+\sum_{j=1}^{m}\sum_{i=0}^{n-j}p_{i,j}=1. \qquad (2)$$

Introducing two functions

$$W_i(j)=(i+1)\prod_{r=0}^{i}\frac{r+\psi_j}{r+1}, \qquad j=\overline{1,m-1}, \qquad i=\overline{0,n-j}; \qquad (3)$$

$$V_i(j)=\frac{\psi_j W_{i-1}(j)}{i-1+\psi_j}, \qquad j=\overline{1,m-1}, \qquad i=\overline{1,n-j},$$

we derive

$$p_{i,j}=\frac{W_i(j)}{i+\psi_j}p_{0,j}, \qquad j=\overline{1,m-1}. \tag{4}$$

Let us

$$x_i=\frac{p_{0,i}}{p_{n,0}},\quad i=\overline{1,m-1} \qquad \text{and} \qquad x_i=\frac{p_{n-i,m}}{p_{n,0}},\quad m\leqq i\leqq n. \tag{5}$$

Introducing probabilities $p_{i,j}$, expressed by $p_{0,j}$ on formula (4) in 2nd–6th group of equations (1) and using expressions (3) and (5) we can derive such a set of linear equations for $x_i(1\leqq i\leqq n)$

$$-W_{n-1}(1)\,x_1+\sum_{j=1}^{m-1}V_{n-j}(j)\,x_j+\psi_m x_{m+1}=-n;$$

for $i=\overline{2,m-1}$ we have

$$\sum_{j=1}^{n-i}V_{n-i-j+1}(j)\,x_j-W_{n-i}(i)\,x_i=0, \qquad \text{if} \quad m>n-i \qquad \text{and}$$

$$\sum_{j=1}^{m-1}V_{n-i-j+1}(j)\,x_j-W_{n-i}(i)\,x_i+\psi_m x_{m+i}=0, \qquad \text{if} \quad m\leqq n-i;$$

for $i=m$ we have (6)

$$\sum_{j=1}^{n-m}V_{n-j-m+1}(j)\,x_j-(n-m+\psi_m)\,x_m=0, \qquad \text{if} \quad m>n-m \qquad \text{and}$$

$$\sum_{j=1}^{m-1}V_{n-j-m+1}(j)\,x_j-(n-m+\psi_m)\,x_m+\psi_m x_{2m}=0, \qquad \text{if} \quad m\leqq n-m;$$

for $i=\overline{m+1,n-1}$ we have

$$\sum_{j=1}^{n-i}V_{n-i-j+1}(j)\,x_j+(n-i+1)\,x_{i-1}-(n-i+\psi_m)\,x_i=0, \qquad \text{if} \quad m>n-i \qquad \text{and}$$

$$\sum_{j=1}^{m-1}V_{n-i-j+1}(j)\,x_j+(n-i+1)\,x_{i-1}-(n-i+\psi_m)\,x_i+\psi_m x_{m+i}=0, \qquad \text{if} \quad m\leqq n-i;$$

for $i=n$ we have

$$-\psi_m x_n+x_{n-1}=0.$$

There are n equations in set (6) and it is much less than the number of equations in the initial set (1). It is possible to use standard routines for solving this set of equations (6) and obtaining x_i, $i=\overline{1,n}$. Let us introduce x_i using (5) in normalizing condition (2), we derive the probability of computer timeout

$$p_{n,0}=\left[1+\sum_{j=1}^{m-1}\sum_{i=0}^{n-j}\frac{W_i(j)\,x_j}{i+\psi_j}+\sum_{j=m}^{n}x_j\right]^{-1}.$$

It is possible using (4) and (5) with x_j and $p_{n,0}$ to derive all probabilities $p_{i,j}$ and basic performance quantities, for example Lq, mean number of waiting requests.

$$Lq = p_{n,0}\left[\sum_{j=1}^{m-1}\sum_{k=1}^{n-j}\frac{kW_{n-k-j}(j)\,x_j}{n-k-j+\psi_j}+\sum_{k=1}^{n-m}kx_{m+k}\right].$$

Let us derive the distribution of the time spent by a request in the system $W(t)$. Using the method of [5] we can derive

$$W(t)=\frac{nQ_{n,0}(t)+\sum_{j=1}^{m-1}\sum_{i=1}^{n-j}\frac{iW_i(j)\,x_j}{i+\psi_j}Q_{i,j}(t)+\sum_{j=m}^{n}(n-j)\,x_jQ_{n-j,m}(t)}{n+\sum_{j=1}^{m-1}\sum_{i=1}^{n-j}\frac{iW_i(j)\,x_j}{i+\psi_j}+\sum_{j=m}^{n}(n-j)\,x_j}, \tag{7}$$

where $Q_{i,j}(t)$ is the probability that the time spent by a request in the system is less than time t, with the condition that there were i requests in the source and j requests on service in the moment of request arrival. If request is arriving when the system is in state $\{n, 0\}$, his time spent is equal to the service time. So $Q_{n,0}(t)=1-\exp(-\mu_1 t)$. For deriving $Q_{i,j}(t)$ let us use the expression for probability $p_k^r(\tau)$ that in the queueing system with finite number of sources during time τ, k requests would arrive on condition that in the begining of time τ there were r requests in the source and during time τ, no request would be serviced [4]

$$p_k^r(\tau)=\frac{r!\,e^{-\lambda r\tau}(e^{\lambda\tau}-1)^k}{k!\,(r-k)!},\qquad 0\leqq k\leqq r. \tag{8}$$

Let a request be arriving when the system is in state $\{i, j\}$ $(j=\overline{1,m}; i=\overline{1,n-j})$. It has rested $(i-1)$ requests in the source, the system has transited in the state $\{i-1, j\}$ and it would be $(n-i-j+1)$ requests in the queue. The choice of requests from the queue on service is accomplished by batch on size m and it would be $q=\left[\frac{n-i-j}{m}\right]$ batches of requests ahead scrutinizing request ([ ] is whole part of the ratio in brackets) and this request is d-th in $(q+1)$ batch $(d=n-i-j+1-qm)$. The time spent by request in the system γ $(\gamma<t)$ is consisting from such parts:

ξ_0 — service time of requests, which already were on service with rate μ_j,

$\sum_{i=0}^{q}\xi_i-\xi_0$ — service time of q batches of requests with size m, which were ahead scrutinizing request with rate of service μ_m of each batch;

ξ_{q+1} — service time of $(q+1)$-th batch of requests, in which there was a scrutinizing request.

Rate of service is μ_c; $c=\min(m, d+f)$, where f is the number of requests which have arrived in the system after scrutinizing request during time $T=\xi_0+\xi_1+\ldots+\xi_q$, and $0 \leqq f \leqq g$, where $g=i-1+\max[j+m(q-1); 0]$. The highest level of f alteration, which is equal to g, is determined by number of requests, which have returned to the source before the begining of service of scrutinizing request.

Let be η_0 requests arrived in the system from the source during time ξ_0 ($0 \leqq \eta_0 \leqq b_0$; $b_0=i-1$). The probability of this event is according with (8) $p_{\eta_0}^{b_0}(\xi_0)$. There has been remained $i-1-\eta_0$ requests in the source. There would be $i-1-\eta_0+j$ requests in the source on the beginning of service of the first batch of requests. The probability of arrival of η_1 requests from the source during time ξ_1 ($0 \leqq \eta_1 \leqq b_1$; $b_1=i-1-\eta_0+j$) is equal $p_{\eta_1}^{b_1}(\xi_1)$. The probability of arrival of η_k requests during time ξ_k ($2 \leqq k < q$; $0 \leqq \eta_k \leqq b_k$; $b_k=i-1+j+(k-1)\,m-\sum_{r=0}^{k-1}\eta_r$) is equal $p_{\eta_k}^{b_k}(\xi_k)$. If rate of service of scrutinizing request would not be depend on the number of arrival requests during time T, then the function $Q_{i,j}(t)$ can be defined as integral of convolution of the distribution function of time ξ_{q+1} with probability density functions of times ξ_k ($k=\overline{0,q}$). Since ξ_{q+1} is dependent on the number of arrival requests during time T, it is necessary every density function of time ξ_k multiply with probability of arrival η_k requests during this time and result summarize on all possible values of η_k ($k=\overline{0,q}$). So

$$
\begin{aligned}
Q_{i,j}(t)= & \sum_{\eta_q=0}^{g-\sum_{r=0}^{q-1}} \sum_{\eta_{q-1}=0}^{b_{q-1}} \cdots \sum_{\eta_1=0}^{b_1} \sum_{\eta_0=0}^{i-1} \int_0^t \int_0^{u_q} \cdots \int_0^{u_2} \int_0^{u_1} [1-e^{-\mu_c}] \times \\
& \times p_{\eta_q}^{b_q}(u_q-u_{q-1})\, d[1-e^{-\mu_m(u_q-u_{q-1})}]\, p_{\eta_{q-1}}^{b_{q-1}}(u_{q-1}-u_{q-2}) \times \\
& \times d[1-e^{-\mu_m(u_{q-1}-u_{q-2})}] \times \ldots \times p_{\eta_0}^{i-1}(u_0)\, d[1-e^{-\mu_j u_0}],
\end{aligned}
\tag{9}
$$

where $u_0=\xi_0$, $u_1=\xi_0+\xi_1$, $\ldots$, $u_q=\sum_{k=0}^{g}\xi_k$.

By substituting expressions for $b_{k,c}$, $p_{\eta_k}^{b_k}(u_k-u_{k-1})$ in (9) and result in (7) we can finally get the expression for distribution function of time spent by request in the system.

Let us derive the Laplace–Stieltjes transform (LST) of the distribution function of $W(t)$

$$
w^*(s)=\frac{\dfrac{n\mu_1}{\mu_1+s}+\sum_{j=1}^{m-1}\sum_{i=1}^{n-j}\dfrac{iW_i(j)\,x_j}{i+\psi_j}\,Q_{i,j}^*(s)+\sum_{j=m}^{n}(n-j)\,x_j\,Q_{n-j,m}^*(s)}{n+\sum_{j=1}^{m-1}\sum_{i=1}^{n-j}\dfrac{iW_i(j)\,x_j}{i+\psi_j}+\sum_{j=m}^{n}(n-j)\,x_j},
\tag{10}
$$

where $Q^*_{i,j}(s)$ is LST of $Q_{i,j}(t)$. Using peculiarity of LST of functions convolution after corresponding transformations we finally derive

$$Q^*_{i,j}(s)=\sum_{\eta_q=0}^{g-\Sigma\eta_r}\ \sum_{\eta_{q-1}=0}^{b_{q-1}}\cdots\sum_{\eta_1=0}^{b_1}\ \sum_{\eta_0=0}^{i-1}\frac{\mu_c}{s+\mu_c}\times$$

$$\times\prod_{k=1}^{q}\left\{\frac{\dfrac{[i-1+j+m(q-k)-\sum_{r=0}^{q-k}\eta_r]!\,\psi_m}{[i-1+j+m(q-k)-\sum_{r=0}^{q-k+1}\eta_r]!}}{\prod_{l=0}^{\eta_{q-k+1}}\left\{\dfrac{s+\mu_m}{\lambda}+\left[i-1+j+m(q-k)-\sum_{r=0}^{q-k+1}\eta_r\right]+l\right\}}\right\}\times$$

$$\times\frac{\dfrac{(i-1)!\,\psi_j}{(i-1-\eta_0)!}}{\prod_{r=0}^{\eta_0}\left[\dfrac{s+\mu_j}{\lambda}+i-1-\eta_0+r\right]}$$

3. Such quantities, which are using for evaluation of queueing systems, as probability of computer utilization $z=(1-p_{n,0})$ (degree of it load) with MCP frames transmission, mean number of waiting requests L_q, can be used as main characteristics of the process of MCP distribution in DNC systems.

The probability of computer utilization with MCP frames distribution and mean number of requests in queue in dependence on machine tools number and channel capacity with transmission time proportional to square root of requests number in batch ($\psi_i=\frac{45}{\sqrt{i}}$; $\lambda=0.1$, $\mu_1=4.5$) are shown in Figs 1 and 2. It appear that with some

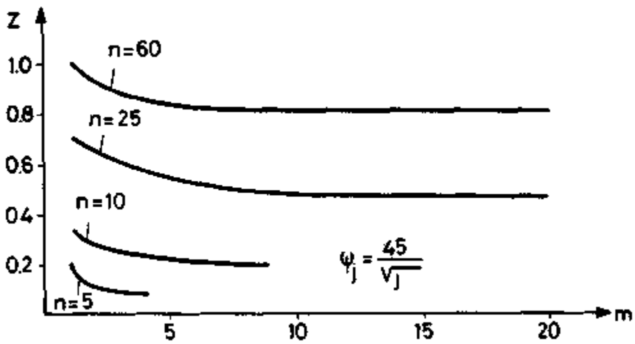

Fig. 1. Probability of computer utilization in dependence on system parameters

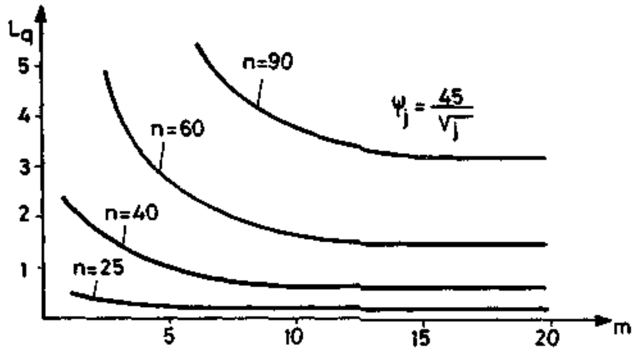

Fig. 2. Mean number of requests in queue in dependence on system parameters

channel capacity and number of machine tools in system, the further growth of the channel capacity do not influence on the DNC system characteristics.

Frames of MCP are transmitted in real time with process rate of machine parts on machine tools. So the probability of MCP transmission without malfunction — R is the main characteristic of DNC system operation. Let us deduce this probability. It is equal to the probability, that time τ from the event of request arrival to the end of frame transmission on this request is less than service time of former request x, which is accidental. Probability that service time of former frame get into segment $[x, x+dx]$ is equal $\lambda e^{-\lambda x}\,dx$. Probability of MCP frame transmission without malfunction during time x is determined by such expression $R_x = \lambda \exp(-\lambda x)\, p\{\tau < x\}\, dx = \lambda \exp(-\lambda x)$ $W(x)\,dx$. So we can derive the general expression for MCP frame transmission without malfunction by integration R_x over x from zero to infinity.

$$R = \int_0^\infty \lambda e^{-\lambda x} W(x)dx = \left[\int_0^\infty e^{-sx}\, dW(x)\right]_{s=\lambda} = w^*(\lambda),$$

where $w^*(\lambda)$ is LST of distribution function defined by expression (10) with $s=\lambda$.

The effect of machine tools number on the probability of MCP transmission without malfunction with highest channel capacity ($m=n$, $\lambda=0.1$; $\mu_1=4.5$) is shown in Fig. 3. The first plot is belonging to the case, when transmission time of batch of requests do not depend on number of requests in batch. The second plot is belonging to the case, when transmission time of batch of requests is proportional to square root of the number of simultaneousily transmitted frames $\left(\psi_i = \frac{45}{\sqrt{i}}\right)$. The third plot is belonging to the case, when transmission time of batch of requests is proportional to the number of simultaneousily transmitted frames $\left(\psi_i = \frac{45}{i}\right)$. It is seen that

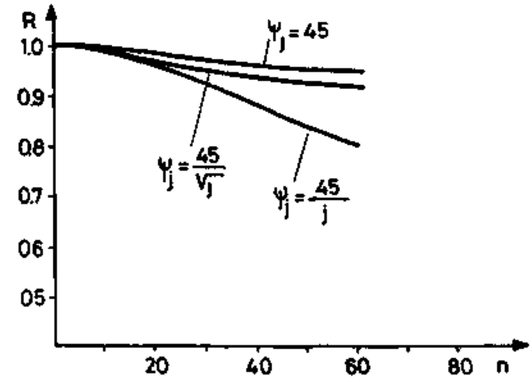

Fig. 3. Effect of machine tools number on the probability of control information transmission without malfunction

dependence of transmission time of batch of requests on the number of simultaneously transmitted frames can in some cases badly influence on the probability of control information transmission without malfunction in DNC system.

4. It is seen that channel capacity (m) and dependens of transmission time of batch of MCP frames on number of simultaneously transmitted frames is badly influenced on the performance quantities of DNC system. As less delivery time dependence on the number of simultaneously transmitted frames, as in better condition the DNC system operate. It is available with invariable parameters of DNC system (number of machine tools, computer load) to choose such channel capacity ($m_* < n$), that main performance quantities would be about their best values, which in general can be achieved when $m = n$.

References

1. *Ratmirov, V. A.*, The fundamentals of program controlled automation for machine tools. Moscow: Mashinostroyeneye, 1978 (in Russian).
2. *Tchernomorov, G. A.*, Analytical modelling of multiprocessor direct numerical control systems with absolute priorities and exponentially distributed service time. Proceeding North Caucases scince centre of higher school. 1979, No. *1*, pp. 49–53 (in Russian).
3. *Kleinrock, L.*, Queueing systems. Vol. I.: Theory. New York–London–Sydney–Toronto: John Wiley & Sons, 1975.
4. *Jaiswell, N. K.*, Priority queues. N.Y. and London: Academic Press, 1968.
5. *Livshitz, B. S., Fidlin, J. V.*, Queueing systems with finite number of sources. Svyaz, Moscow, 1968 (in Russian).

Вероятностный анализ процесса выдачи управляющей информации для систем группового управления

А. Л. ЛИТВИНОВ

(Харьков)

Создание на базе станков с числовым программным управлением (ЧПУ) систем группового управления (СГУ) оборудованием от ЭВМ является важным направлением автоматизации производственных процессов. ЭВМ в таких системах связана с отдельными устройствами ЧПУ (УЧПУ) станков через канал связи и выдача кадров управляющей информации по запросам отдельных УЧПУ является важнейшей задачей, решаемой в СГУ. В качестве моделей СГУ можно использовать замкнутые системы массового обслуживания. Выдача отдельных кадров управляющей информации осуществляется с определенным тактом, поэтому имеется возможность организовать одновременную раздачу кадров информации нескольким УЧПУ в режиме разделения времени. Время выдачи группы кадров управляющей информации может зависеть от количества кадров в группе и замкнутая система массового обслуживания, в которой обслуживание запросов осуществляется группами с переменной скоростью, может быть использована в качестве модели подобных систем. Количество одновременно выдаваемых кадров информации m определяется пропускной способностью канала связи и $m \leqq n$, где n — количество станков в системе. Выведена система из 0,5 m $(2n+1-m)+1$ уравнений, описывающая функционирование этой системы в предположении пуассоновских потоков событий. Эта система уравнений сведена к системе из n уравнений, для решения которой можно использовать стандартные подпрограммы, входящие в состав программного обеспечения современных ЭВМ. Получены следующие характеристики функционирования СГУ: вероятность простоя, среднее число запросов в очереди. Выведена вероятность передачи кадров управляющей информации без сбоя. Эта вероятность может использоваться в качестве основного критерия функционирования СГУ. Приведены графики основных характеристик системы в зависимости от её параметров.

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